CALCULATION OF RADIATIVE COOLING OF HEATING ELEMENTS

V. V. Ivanov

Inzhenerno-Fizicheskii Zhurnal, Vol. 11, No. 4, pp. 542–544, 1966

UDC 621.039

An approximate solution is obtained for the problem of the unsteady temperature field in solids with internal heaters cooled by radiation.

It is known that the heat release from the surface of many heating elements operating at low pressures is accomplished predominantly by radiation. To find the temperature field, which determines the safe limits of thermal loading, in these conditions, we require the integration of the differential equation of heat conduction

$$\frac{\partial \Theta(X, \mathrm{Fo})}{\partial \mathrm{Fo}} = \nabla^2 \Theta(X, \mathrm{Fo}) + \mathrm{Po} \qquad (1)$$

with a nonlinear boundary condition expressed by the Stefan-Boltzmann law

$$-\frac{\partial \Theta(1, \mathrm{Fo})}{\partial X} = \mathrm{Sk}\,[\Theta^4(1, \mathrm{Fo}) - 1], \qquad (2)$$

the condition of symmetry

$$\frac{\partial \Theta(0, \mathrm{Fo})}{\partial X} = 0 \qquad (3)$$

and the initial condition

$$\Theta(X, 0) = \Theta_0. \qquad (4)$$

Here

$$0 \leqslant X = x/R \leqslant 1, \quad 0 \leqslant \mathrm{Fo} = a\tau/R^2 < \infty,$$

$$\Theta(X, \mathrm{Fo}) = T/T_c, \quad \mathrm{Sk} = \sigma_b T_c^3 R/\lambda, \quad \mathrm{Po} = q_v R^2/\lambda T_c,$$

$$\nabla^2 = \frac{\partial^2}{\partial X^2} + \frac{\xi}{X}\frac{\partial}{\partial X},$$

and ξ is a coefficient which is equal to 0, 1, and 2 in the plane, cylindrical, and spherical problems, respectively.

By putting

$$\Theta(X, \mathrm{Fo}) = U(X, \mathrm{Fo}) + t(X) \quad K, \qquad (5)$$

where the function $t(X)$ is a solution of the problem (1)–(4) when $\mathrm{Fo} \to \infty$, i.e., the steady temperature

$$t(X) = \frac{\mathrm{Po}}{2\xi + 2}(1 - X^2) + K,$$

$$K = [1 + \mathrm{Po}(\xi + 1)\mathrm{Sk}]^{1/4},$$

we obtain a system of equations for the determination of $U_*(X, \mathrm{Fo}) = U(X, \mathrm{Fo})/K$:

$$\frac{\partial U_*(X, \mathrm{Fo})}{\partial \mathrm{Fo}} = \nabla^2 U_*(X, \mathrm{Fo}), \qquad (6)$$

$$\frac{\partial U_*(1, \mathrm{Fo})}{\partial X} = \mathrm{Sk}_*[1 - U_*^4(1, \mathrm{Fo})], \quad \mathrm{Sk}_* = K^3\mathrm{Sk}, \qquad (7)$$

$$\frac{\partial U_*(0, \mathrm{Fo})}{\partial X} = 0, \qquad (8)$$

$$U_*(X, 0) = \frac{1}{K}\left[\Theta_0 - \frac{\mathrm{Po}}{2\xi + 2}(1 - X^2)\right] = U_{*0}(X). \qquad (9)$$

In order to find the unsteady temperature $U_*(X, \mathrm{Fo})$, we employ the substitution

$$-\frac{\ln W(X, \mathrm{Fo})}{p} =$$

$$= \frac{1}{2}[\mathrm{Arth}\,U_*(X, \mathrm{Fo}) + \mathrm{arctg}\,U_*(X, \mathrm{Fo})], \qquad (10)$$

in which p is a parameter.

The function $W(X, \mathrm{Fo})$ linearizes the boundary condition (7), reducing it to the form of a boundary condition of the 3rd kind:

$$\frac{\partial W(1, \mathrm{Fo})}{\partial X} = -p\,\mathrm{Sk}_*\,W(1, \mathrm{Fo}). \qquad (11)$$

The symmetry condition (8) does not change,

$$\frac{\partial W(0, \mathrm{Fo})}{\partial X} = 0, \qquad (12)$$

the initial condition (9) is written in the form:

$$W(X, 0) =$$

$$= \exp\left\{-\frac{p}{2}[\mathrm{Arth}\,U_{*0}(X) + \mathrm{arctg}\,U_{*0}(X)]\right\}, \qquad (13)$$

and the differential equation of heat conduction takes the form

$$\frac{\partial W(X, \mathrm{Fo})}{\partial \mathrm{Fo}} = \nabla^2 W(X, \mathrm{Fo}) + f(X, \mathrm{Fo}), \qquad (14)$$

$$f(X, \mathrm{Fo}) = p\left[\frac{\partial U_*}{\partial X} \Big/ (1 - U_*^4)\right]^2 (4U_*^3 - p)\,W. \qquad (15)$$

We note that since Arth α exists when $|\alpha| < 1$, the substitution (10) may be employed in the case when $|U_{*0}(X)| < 1$.

The quantity $f(X, \mathrm{Fo})$ in Eq. (14), an internal heat source of variable power, is equal to zero when $X = 0$ (condition (8)) and reaches its maximum value at the surface. By appropriate choice of the parameter p, the nonlinear complex (15) may be reduced to such a degree that it practically ceases to have an influence on the temperature field $W = W(X, \mathrm{Fo})$.

Since $f(X, \mathrm{Fo}) \to 0$ when $4U_*^3 \to p$ and $U_*(X, \mathrm{Fo})$ changes throughout the process from $U_{*0}(X)$ with $\mathrm{Fo}^* = 0$ to 1.0 with $\mathrm{Fo} \to \infty$, we divide the region of temperature variation $U_*(X, \mathrm{Fo})$ into a series of intervals $(U_{*0} - U_{*1}, \ldots, U_{*i-1} - U_{*i}, \ldots)$, and choose p_i for each interval according to the relation

$$p_i = 4U_{*i}^3.$$

Then the temperature distribution for the i-th interval is used as the initial condition for the next interval, and so on.

Such a choice of the parameter p_i leads to the fact that $f(X, Fo) \approx 0$. In this case the temperature $U_*(X, Fo)$ is found on the basis of the solution of the problem (11)–(15) with $f(X, Fo) = 0$, as presented in [1], and of relation (10), as constructed in [2].

Data reduction on a computer has allowed us to establish that for the case $\xi = 0$, $|1 - U_{*0}(X) \leq 0.85$, the error in calculating $U_*(X, Fo)$ does not exceed 5% for all X, if the number of intervals is equal to one when $Sk_* < 1.0$, two when $1.0 \leq Sk_* < 2.0$, three when $2.0 \leq Sk_* < 3.0$, and four when $3.0 \leq Sk_* < 4.0$. With increase of ξ, and also with decrease of $|1 - U_{*0}|$, the accuracy of the calculation increases. Increase of the number of intervals leads to a reduction in the value of $f(X, Fo)$, and to an increase in the accuracy of determination of $U_*(X, Fo)$.

The results obtained here may be used to calculate the temperature fields in heating elements cooled by radiation, as well as to estimate the times needed to heat separate layers to a given temperature.

REFERENCES

1. A. V. Luikov and Yu. A. Mikhailov, Theory of Heat and Mass Transfer [in Russian], Gosenergoizdat, 1963.

2. V. V. Ivanov and Yu. V. Vidin, Izv. VUZ. Chernaya metallurgiya, no. 5, 1965.

22 January 1966

Institute of Electrical Engineering, Novosibirsk